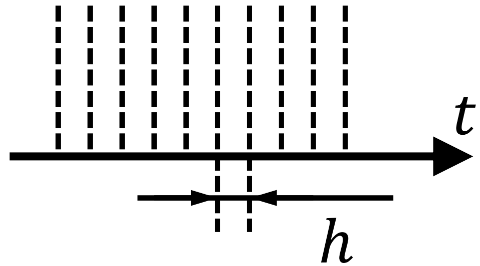
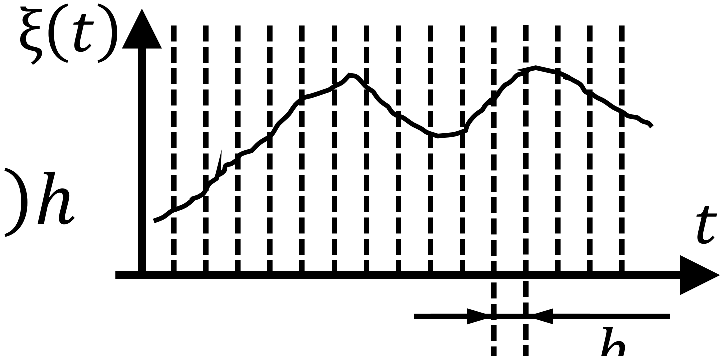
$$\xi(t) \Rightarrow \xi(jh) \equiv \xi(j)$$



$$Ш(t) = \sum_{j=-\infty}^{\infty} \delta\,(t - jh)$$



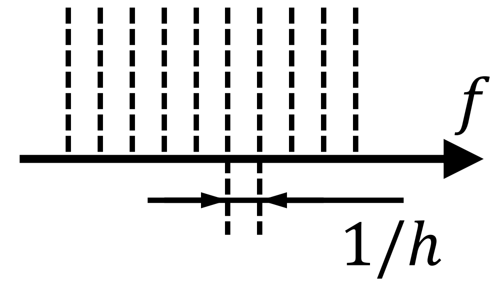
$$\xi(j) = \xi(t)Ш(t)h$$

$$\xi(j) \Rightarrow S_h(f) \qquad S_h(f) = S(f) \otimes Ш^{\mathcal{F}}(f)h$$

$$Ш(t) = \sum_{n=-\infty}^{\infty} d_n e^{-if_n t} = \sum_{n=-\infty}^{\infty} d_n e^{-i\frac{2\pi}{h}nt}$$

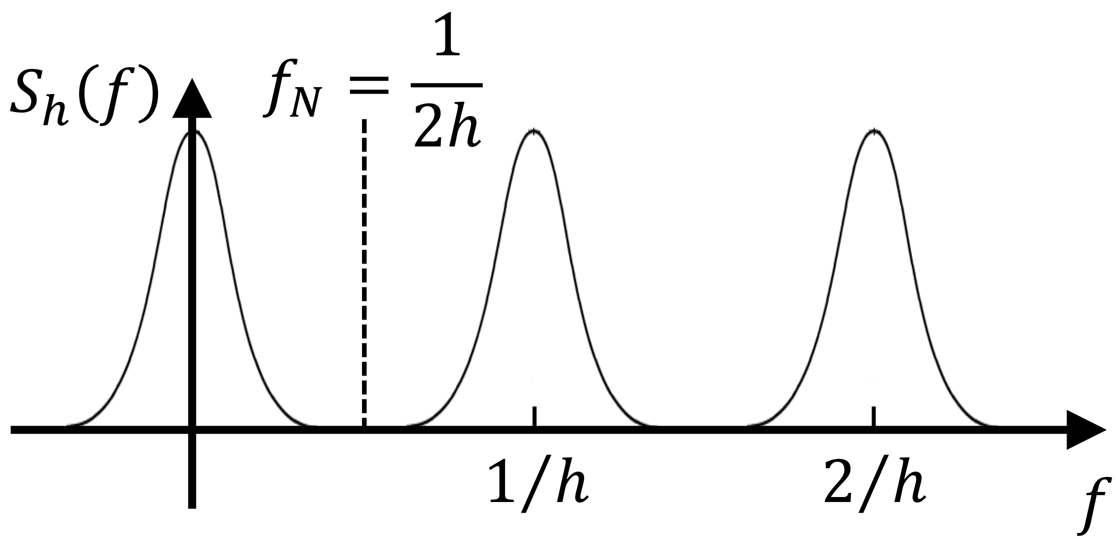
$$d_n = \frac{1}{h}\int_{-h/2}^{h/2} \delta(t - jh)\, e^{-i\frac{2\pi}{h}nt} dt = \frac{1}{h} e^{-i\frac{2\pi}{h}njh} = \frac{1}{h}$$

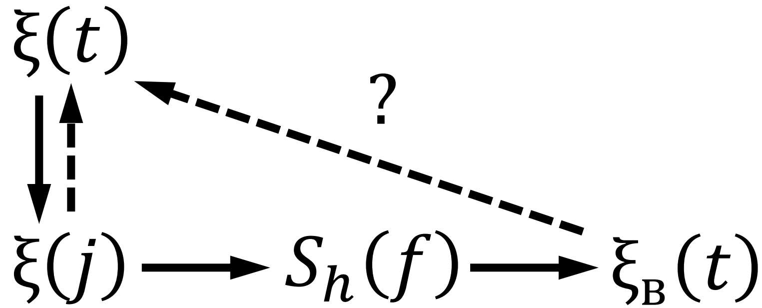
$$Ш^{\mathcal{F}}(f) = \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{h}\right)\frac{1}{h}$$



$$S_h(f) = S(f) \otimes Ш^{\mathcal{F}}(f)h =$$

$$= h \int_{-\infty}^{\infty} S(\vartheta) \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{h} - \vartheta\right)\frac{1}{h}\, d\vartheta = \sum_{n=-\infty}^{\infty} S\left(f - \frac{n}{h}\right)$$





$$S(f) \Rightarrow f_{\max} < f_{\mathrm{N}}$$

$$S(f) = S_h(f) \cdot \Pi^{\mathcal{F}}(f) \qquad \Pi^{\mathcal{F}}(f) = \begin{cases} 1, f \in [-f_{\mathrm{N}}, f_{\mathrm{N}}] \\ 0, |f| > f_{\mathrm{N}} \end{cases}$$

$$\xi_{\mathrm{B}}(t) = \mathcal{F}\left(S_h(f) \cdot \Pi^{\mathcal{F}}(f)\right)$$

$$\xi_{\mathrm{B}}(t) = \xi(j) \otimes \Pi(t)$$

$$\Pi(t) = \int_{-\infty}^{\infty} \Pi^{\mathcal{F}}(f) e^{i2\pi f t} df =$$

$$= \int_{-f_N}^{f_N} e^{i2\pi f t} df = \frac{1}{i2\pi t}\left(e^{i2\pi f_N t} - e^{-i2\pi f_N t}\right) =$$

$$= \frac{1}{i2\pi t} 2i\sin(2\pi f_N t) = \frac{1}{\pi t}\sin\left(\frac{\pi t}{h}\right) = \frac{1}{h}\operatorname{sinc}\left(\frac{\pi t}{h}\right)$$

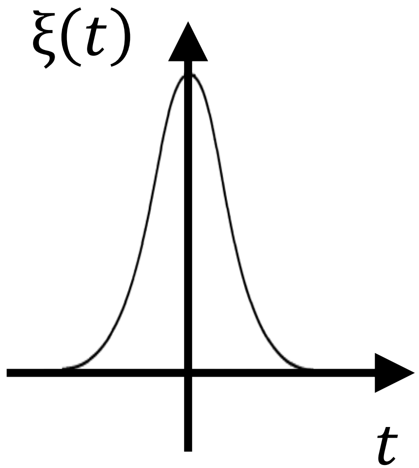
$$\xi_{\text{в}}(t) = \xi(j) \otimes \Pi(t)$$

$$\xi(j) = \xi(t)h \sum_{j=-\infty}^{\infty} \delta\,(t - jh)$$

$$\xi_{\text{в}}(t) = h \sum_{j=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{h} \text{sinc}\left(\frac{\pi\vartheta}{h}\right) \xi(t - \vartheta) \delta(\text{t} - \text{jh} - \vartheta) d\vartheta =$$

$$= \sum_{j=-\infty}^{\infty} \xi\,(jh) \text{sinc}\left(\frac{\pi(t - jh)}{h}\right) =$$

$$= \sum_{j=-\infty}^{\infty} \xi\,(jh) \frac{\sin\left(\frac{\pi(t - jh)}{h}\right)}{\frac{\pi(t - jh)}{h}}$$

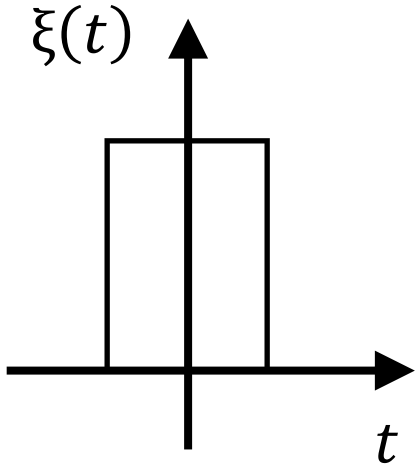


$$\Delta\omega\tau \approx 3.7$$

$$\Delta f\tau \approx \frac{3.7}{2\pi} \approx 0.6$$

$$\tau = 1\text{с} \;\Rightarrow\; \Delta f \approx 0.6\text{Гц} \;\Rightarrow\; f_N = 5\Delta f \approx 3\text{Гц}$$

$$\frac{1}{2h} \approx 3\frac{1}{\text{с}} \;\Rightarrow\; h = \frac{1}{6}\text{с} \;\Rightarrow\; N = 6$$



$$\Delta\omega\tau \approx 12$$

$$\Delta f \tau \approx \frac{12}{2\pi} \approx 2$$

$$\tau = 1\text{с} \quad \Rightarrow \quad \Delta f \approx 2\text{Гц} \quad \Rightarrow \quad f_N = 5\Delta f \approx 10\text{Гц}$$

$$\frac{1}{2h} \approx 10\,\frac{1}{\text{с}} \quad \Rightarrow \quad h = \frac{1}{20}\,\text{с} \quad \Rightarrow \quad N = 20$$

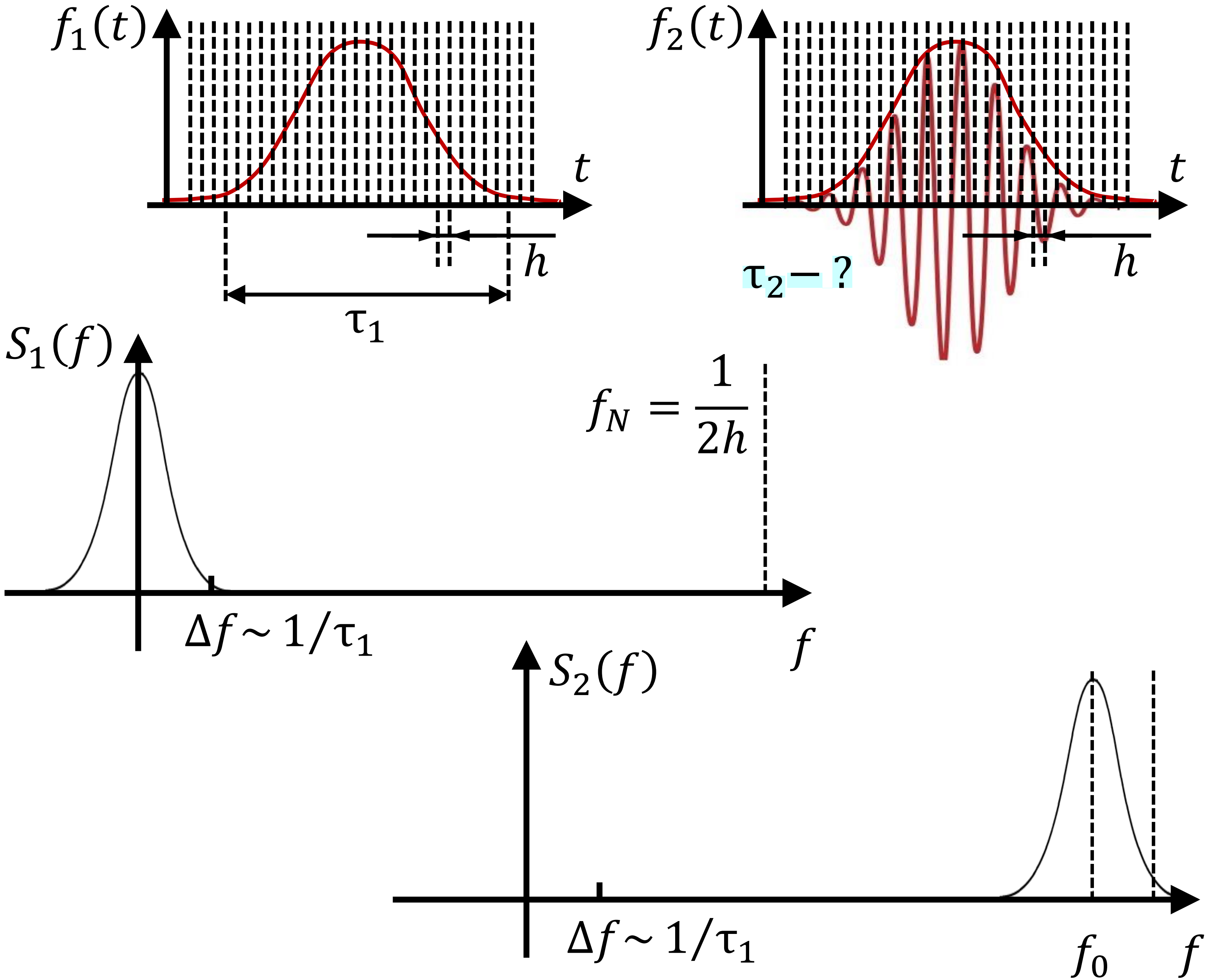
Дискретное преобразование Фурье (ДПФ)

$$\xi(j) = \sum_{n=0}^{N-1} S(n) e^{i\frac{2\pi nj}{N}} \qquad S(n) = \frac{1}{N}\sum_{j=0}^{N-1} \xi(j) e^{-i\frac{2\pi nj}{N}}$$

$$\xi(t) \Rightarrow \xi(j) \equiv \xi(jh) \qquad S(f) \Rightarrow S(n) \equiv S(n\Delta f)$$

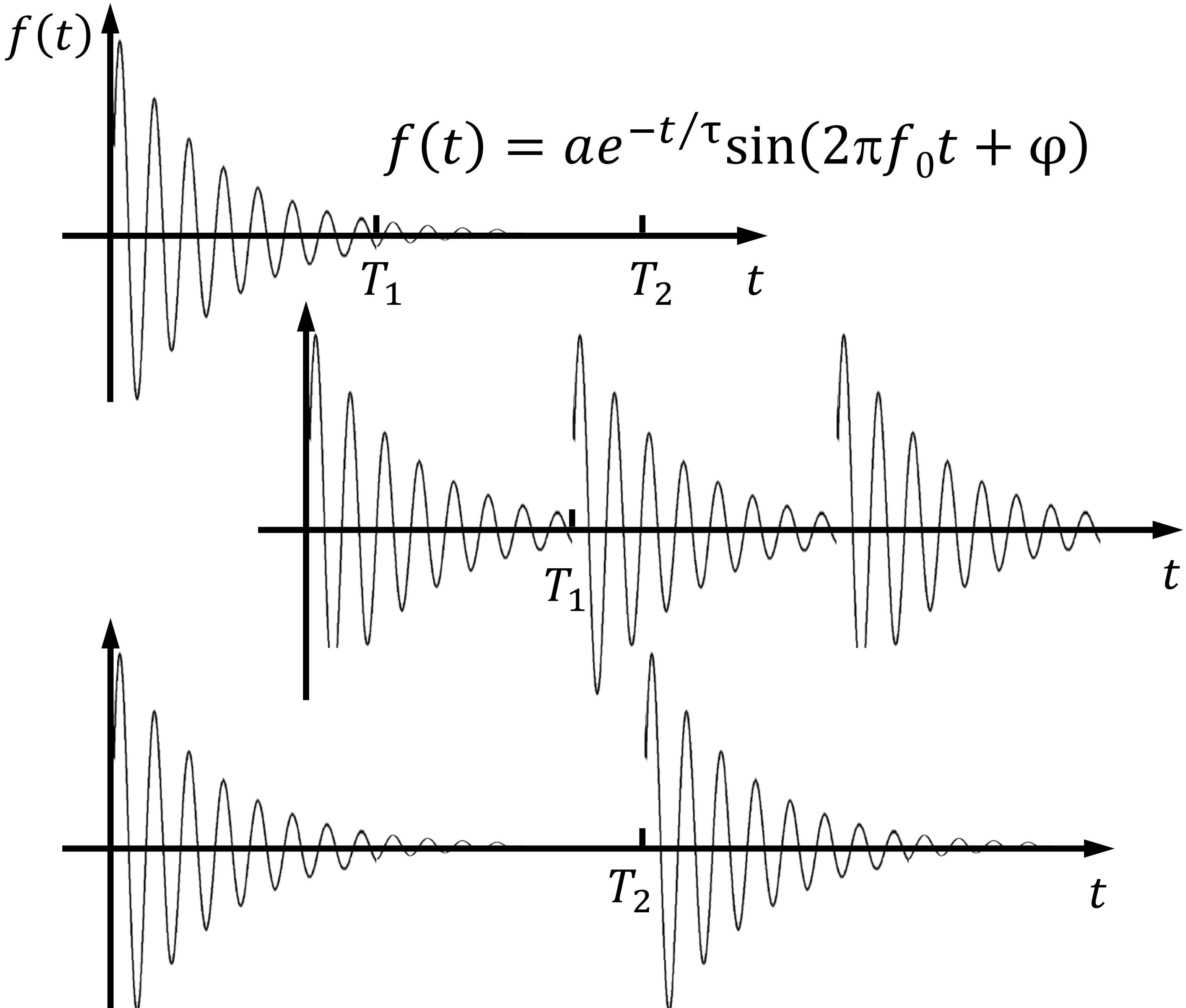
$$T = \frac{1}{\Delta f} \qquad F = \frac{1}{h}$$

$$N = \frac{T}{h} = \frac{F}{\Delta f} = \frac{1}{h} : \frac{1}{T} = \frac{T}{h}$$



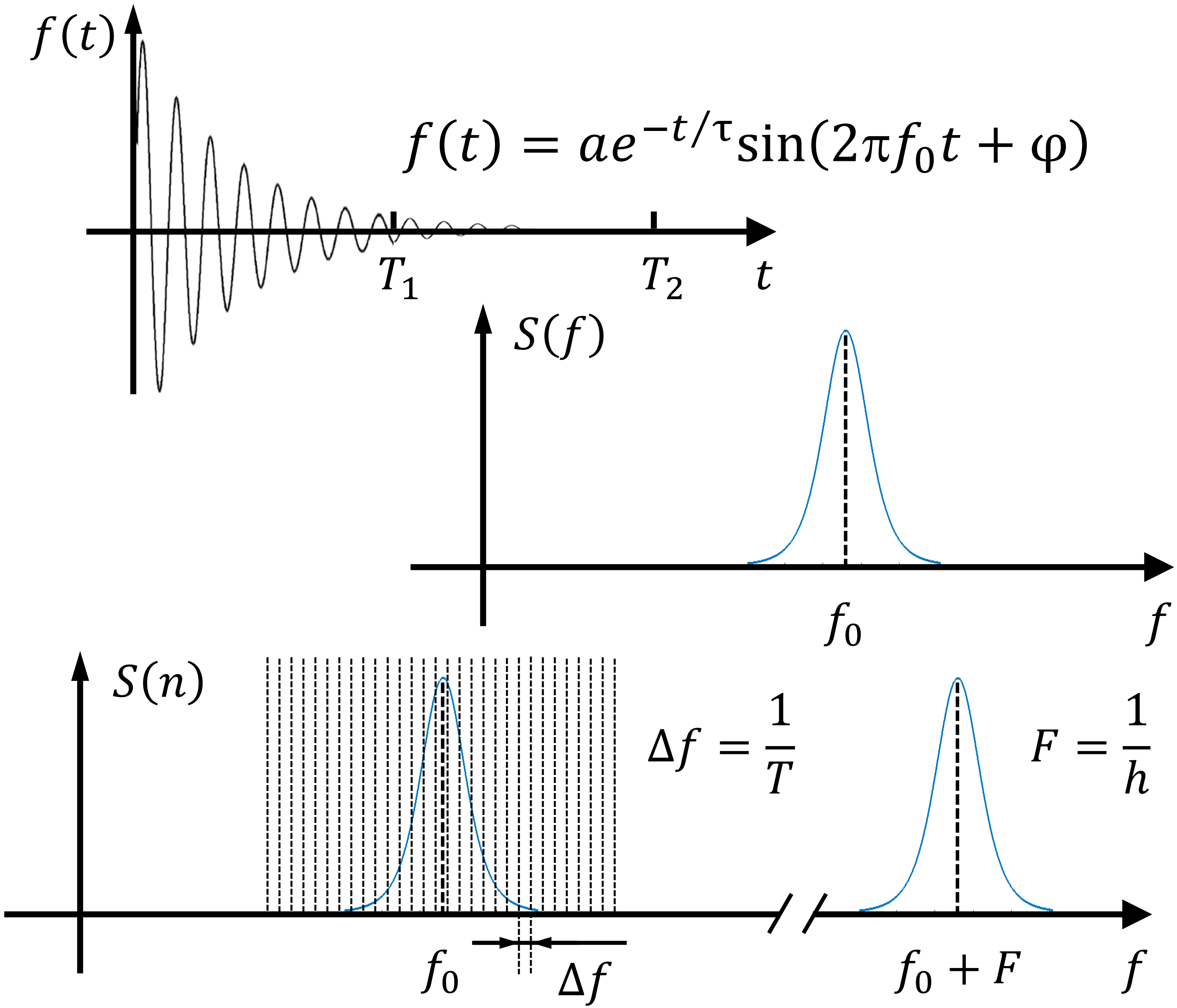
$f_1(t)$
t
h
τ_1
$f_2(t)$
t
$\tau_2 - ?$
h
$S_1(f)$
$f_N = \frac{1}{2h}$
$\Delta f \sim 1/\tau_1$
f
$S_2(f)$
$\Delta f \sim 1/\tau_1$
f_0
f

Важность интервала периодизации (ДПФ)



Важность интервала периодизации (ДПФ)

$$f(t) = ae^{-t/\tau}\sin(2\pi f_0 t + \varphi)$$



$$\Delta f = \frac{1}{T}$$

$$F = \frac{1}{h}$$

Быстрое преобразование Фурье (БПФ)

$$\xi(j) = \sum_{n=0}^{N-1} S(n) e^{i\frac{2\pi n j}{N}} \qquad S(n) = \frac{1}{N}\sum_{j=0}^{N-1} \xi(j) e^{-i\frac{2\pi n j}{N}}$$

число операций (ЧО) $= N^2$

$$\xi(j) \quad j = 0, 1, 2, \dots, N-1$$

$$\xi_1(j) \equiv \xi(j = 2m) \qquad \xi_2(j) \equiv \xi(j = 2m + 1)$$

$$S(n) = \frac{1}{N}\sum_{j=0}^{N-1} \xi(j) e^{-i\frac{2\pi n j}{N}}$$

$$NS(n) = \sum_{j=0}^{N-1} \xi(j) W_N^{-nj} =$$

$$= \sum_{m=0}^{(N/2)-1} \xi(2m) W_N^{-2nm} + \sum_{m=0}^{(N/2)-1} \xi(2m+1) W_N^{-2nm} W_N^{-n}$$

$$W_N^{-2nm} = e^{-i\frac{2\pi}{N}2nm} = e^{-i\frac{2\pi}{(N/2)}nm} = W_{N/2}^{-nm}$$

$$NS(n) = \sum_{j=0}^{N-1} \xi(j) W_N^{-nj} =$$

$$= \sum_{m=0}^{(N/2)-1} \xi(2m) W_{N/2}^{-nm} + W_N^{-n} \cdot \sum_{m=0}^{(N/2)-1} \xi(2m+1) W_{N/2}^{-nm}$$

$$\downarrow \qquad n = \{0 \ldots N\} \qquad \downarrow$$

$$\frac{N}{2} S_1(n) \qquad\qquad \frac{N}{2} S_2(n)$$

$$ЧО_{1,2} = N \times \left(\frac{N}{2}\right) = \frac{N^2}{2}$$

$$ЧО = \frac{N^2}{2} + \frac{N^2}{2} = N^2$$

$$NS(n + N/2) = \sum_{j=0}^{N-1} \xi(j) W_N^{-(n+N/2)j} =$$

$$= \sum_{m=0}^{(N/2)-1} \xi(2m) W_{N/2}^{-(n+N/2)m} + W_N^{-(n+N/2)}$$

$$\cdot \sum_{m=0}^{(N/2)-1} \xi(2m+1) W_{N/2}^{-(n+N/2)m}$$

$$W_N^{-(n+N/2)} = e^{-i\frac{2\pi}{N}(n+N/2)} = e^{-i\frac{2\pi}{N}(n+N/2)} = e^{-i\pi} e^{-i\frac{2\pi}{N}n} =$$

$$= -e^{-i\frac{2\pi}{N}n} = -W_N^{-n}$$

$$W_{N/2}^{-(n+N/2)m} = W_{N/2}^{-nm}$$

$$NS(n)=\sum_{j=0}^{N-1}\xi(j)W_N^{-nj}=$$

$$=\sum_{m=0}^{(N/2)-1}\xi(2m)W_{N/2}^{-nm}+W_N^{-n}\cdot\sum_{m=0}^{(N/2)-1}\xi(2m+1)W_{N/2}^{-nm}$$

$$n=\{0\ldots N/2\}$$

$$\frac{N}{2}S_1(n) \qquad \frac{N}{2}S_2(n)$$

$$ЧО_{1,2}=\frac{N}{2}\times\left(\frac{N}{2}\right)=\frac{N^2}{4}$$

$$ЧО=\frac{N^2}{4}+\frac{N^2}{4}=\frac{N^2}{2}$$

$$N = 2^M$$

$$\text{ЧО}_{\text{БПФ}} = N\log_2 N$$

$$N = 2^{10} \quad \Rightarrow \quad \frac{\text{ЧО}_{\text{ДПФ}}}{\text{ЧО}_{\text{БПФ}}} = \frac{N^2}{N\log_2 N} \approx 100$$